On the possibility of level broadening in a quantum dot due to electrostatic interaction with a gate electrode

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In this article, we consider a quantum dot system which is interacting with a spatially separated metallic gate electrode via direct Coulomb interaction. Here, the gate electrode is described by an idealized two-dimensional electron gas. Due to Coulomb scattering effects, the latter may introduce level broadening to the quantum dot system.

I. INTRODUCTION

Metallic gate electrodes constitute a common experimental means to control the energy spectrum of quantum dot structures[1]. Electron charges inside the quantum dot typically induce image charges within the gate electrodes. Obviously, such an electrostatic influence provides an energetical shift of the quantum dot states. However, due to Coulomb scattering processes between quantum dot electrons and electrons inside the dissipative gate electrode, the dot system in general will be subject to level broadening as well. In other words, the "dissipation" or "friction" of gate electrons becomes visible to the quantum dot, analogous to a Coulomb drag effect[2]. In this article, we discuss an idealized model for the modified spectral properties of a quantum dot under the influence of the Coulomb interaction with a two-dimensional electron gas (2DEG). Finally, the possible consequences for qubit systems[9,10] are discussed.

II. THE MODEL

Fig. 1 shows a schematic sketch of the considered system, consisting of a quantum dot and a 2DEG which represents a normal metallic gate electrode. In a different context, a similar system has been discussed by Kato et al.[3,4,5]. The two spatially separated (i.e. non-overlapping) subsystems interact with each other via direct electrostatic Coulomb interaction.

The model Hamiltonian for the interacting quantum dot system and the Fermi gas of the gate electrode reads as

$$H = \sum_i E_i a_i^\dagger a_i + \sum_k \epsilon_k b_k^\dagger b_k + \sum_{i,i',k,k'} V_{ikk'i'}\ a_i^\dagger b_k^\dagger b_{k'} a_{i'},$$

$$(1)$$

with wavevector indices k, k'. (The discrete sum over k can be considered as the result of periodic boundary conditions for the Fermi gas states, however, can eventually be replaced by a k-integral in the thermodynamic limit.) a and b are the quasi-particle annihilation operators for the quantum dot and the reservoir, respectively. E and ϵ denote the corresponding energies, whereas V is the Coulomb matrix for the direct electrostatic inter-
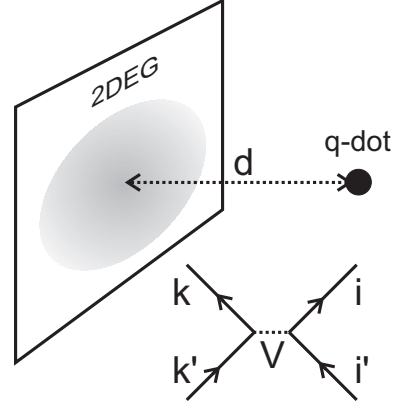

FIG. 1: Quantum dot system and gate (2DEG) with electrostatic interaction (V). (The shaded area corresponds to the induced image charge.)

action between the quantum dot and the spatially separated reservoir (i.e., without electron exchange between the two subsystems due to vanishing overlap of the latter). For simplicity, intra-dot interaction and the spin degree of freedom will not be considered in this paper. Furthermore, transitions between the quantum dot states and the Fermi gas are assumed to be negligible (due to a sufficient spatial separation).

III. FUNDAMENTAL ASPECTS

In order to understand the main physical effect, we want to focus on a single quantum dot level E_0 in this section. The Hamiltonian thus reads as

$$H = E_0 a^\dagger a + \sum_k \epsilon_k b_k^\dagger b_k + \sum_{k,k'} v_{kk'}\ a^\dagger a b_k^\dagger b_{k'}, \quad (2)$$

where $a \equiv a_0$ and $v_{kk'} \equiv V_{0kk'0}$. The quantum dot occupation number operator $n = a^\dagger a$ has the eigenvalues 0 and 1. This motivates us to rewrite H as:

$$H = (1-n) \cdot \left(\sum_k \epsilon_k b_k^\dagger b_k \right) \qquad (3)$$

$$+ n \cdot \left(E_0 + \sum_k \tilde{\epsilon}_k \tilde{b}_k^\dagger \tilde{b}_k \right),$$

where we have diagonalized the hermitian matrix

$$\hat{\epsilon}_{kk'} = \delta_{kk'}\epsilon_k + v_{kk'} \tag{4}$$

by use of a unitary single-particle transformation U such that

$$\hat{\epsilon}_{kk'} = \sum_j U_{kj}\tilde{\epsilon}_j U^{\dagger}_{jk'} \tag{5}$$

with eigenvalues $\tilde{\epsilon}_k$. Furthermore, we have introduced the transformed electron operators

$$\tilde{b}_j = \sum_k U^{\dagger}_{jk} b_k. \tag{6}$$

Since the Hamiltonian H is diagonal with respect to the occupation number n eigen-subspaces, we obtain

$$H = \sum_k \epsilon_k n_k \quad \text{for} \quad n = 0, \tag{7}$$

$$H = \sum_k \tilde{\epsilon}_k \tilde{n}_k + E_0 \quad \text{for} \quad n = 1, \tag{8}$$

where $n_k = b^{\dagger}_k b_k$ and $\tilde{n}_k = \tilde{b}^{\dagger}_k \tilde{b}_k$ are the occupation number operators for 2DEG states. These two subspace Hamiltonians are trivial, and the many-body eigenstates can readily be formulated as Slater determinants:

$$|J\rangle = b^{\dagger}_{k(J)_1} \cdots b^{\dagger}_{k(J)_N} |vac\rangle, \tag{9}$$

$$|K\rangle = a^{\dagger}\tilde{b}^{\dagger}_{k(K)_1} \cdots \tilde{b}^{\dagger}_{k(K)_N} |vac\rangle, \tag{10}$$

for a Slater determinant $|J\rangle$ with $\langle J|n|J\rangle = 0$ and $|K\rangle$ with $\langle K|n|K\rangle = 1$. Here, N denotes the number of 2DEG electrons in the states $|J\rangle$ and $|K\rangle$, and $(k_1,\ldots,k_N)$ uniquely identifies the occupied single-particle states within a Slater determinant (with index order $k_1 < \cdots < k_N$). Since we have to consider a grandcanonical ensemble, all possible Slater determinants with all possible N of the given type are allowed. As for the eigenenergies of the eigenstates $|J\rangle, |K\rangle$ we obtain

$$E_J = E^0_J, \tag{11}$$
$$E_K = E^1_K + E_0, \tag{12}$$

with the 2DEG energies

$$E^0_J = \sum_k n(J)_k \, \epsilon_k, \tag{13}$$

$$E^1_K = \sum_k \tilde{n}(K)_k \, \tilde{\epsilon}_k, \tag{14}$$

where $n(J)_k$ and $\tilde{n}(K)_k$ denote the occupation numbers in the Slater determinants for the many-body indices J and K, respectively. One has to note that due to the interaction between the quantum dot and the gate, the subspaces for $n = 0$ and $n = 1$ in general have two different (not trivially overlapping) single-particle eigenbases for the construction of Slater determinants as many-body eigenstates. This property will turn out to be responsible for level broadening in the quantum dot.

We now want to consider the spectral function $A(\omega)$ (density of states) for the *quantum dot* state. This quantity can be derived from the retarded two-point Green's function of the system[6,7,8]. Within the scope of the many-body eigenbasis representation, we can directly employ the Lehmann representation[7] of A:

$$A(\omega) = -2\,\text{Im}\left[\sum_{\substack{J \\ \langle J|n|J\rangle=0}} \sum_{\substack{K \\ \langle K|n|K\rangle=1}} \right. \tag{15}$$
$$\frac{\hbar}{\hbar\omega - E_0 - (E^1_K - E^0_J) + i\eta}$$
$$\times (w_J + w_K)$$
$$\left. \times \langle J|a|K\rangle\langle K|a^{\dagger}|J\rangle \right]$$

$$= 2\pi\hbar \sum_{\substack{J \\ \langle J|n|J\rangle=0}} \sum_{\substack{K \\ \langle K|n|K\rangle=1}} \tag{16}$$
$$\delta_\eta(\hbar\omega - E_0 - (E^1_K - E^0_J))$$
$$\times (w_J + w_K) \, |\langle J|a|K\rangle|^2,$$

with

$$\delta_\eta(x) = \frac{1}{\pi}\frac{\eta}{x^2 + \eta^2}, \tag{17}$$

where $\eta \to 0+$ (after the thermodynamic limit for the sum over 2DEG states). From the matrix element of a, one can see that only the combinations $\langle J|n|J\rangle = 0$ and $\langle K|n|K\rangle = 1$ with the same N for J and K can provide non-vanishing terms. $w \geq 0$ are the eigenvalues of the many-body statistical operator

$$\rho = \sum_{\substack{J \\ \langle J|n|J\rangle=0}} w_J|J\rangle\langle J| + \sum_{\substack{K \\ \langle K|n|K\rangle=1}} w_K|K\rangle\langle K|, \tag{18}$$

with grandcanonical equilibrium form

$$w_J = \frac{1}{Z}\exp\left(-\beta(E^0_J - \mu\sum_k n(J)_k)\right), \tag{19}$$

$$w_K = \frac{1}{Z}\exp\left(-\beta(E_0 - \mu + E^1_K - \mu\sum_k \tilde{n}(K)_k)\right) \tag{20}$$

where $\beta = 1/(k_B T)$ and μ denotes the chemical potential, and Z is the grandcanonical partition function for all J, K such that $\sum w = 1$. This equilibrium condition corresponds to the assumption of relaxation processes within the gate (which of course are not explicitly considered in H).

The many-body matrix element $\langle J|a|K\rangle$ can be evaluated by use of the single-particle transformation U in Eq. (6):

$$\langle J|a|K\rangle = \langle vac| \left(b^{\dagger}_{k(J)_1} \cdots b^{\dagger}_{k(J)_N}\right)^{\dagger} \tag{21}$$

$$\times \tilde{b}^{\dagger}_{k(K)_1} \cdots \tilde{b}^{\dagger}_{k(K)_N} |vac\rangle$$

$$= \langle vac|b_{k(J)_N} \cdots b_{k(J)_1} \qquad (22)$$

$$\times \tilde{b}^{\dagger}_{k(K)_1} \cdots \tilde{b}^{\dagger}_{k(K)_N} |vac\rangle$$

$$= \sum_{l_1,...,l_N} U_{l_1 k(K)_1} \cdots U_{l_N k(K)_N} \qquad (23)$$

$$\langle vac|b_{k(J)_N} \cdots b_{k(J)_1} b^{\dagger}_{l_1} \cdots b^{\dagger}_{l_N}|vac\rangle.$$

By use of $\{b_j, b^{\dagger}_k\} = \delta_{jk}$ and $b_k|vac\rangle = 0$ we finally obtain

$$|\langle J|a|K\rangle|^2 = \Big| \sum_{l \in Perm(k(J))} (-1)^{P(l,k(J))} \qquad (24)$$

$$\times U_{l_1 k(K)_1} \cdots U_{l_N k(K)_N} \Big|^2,$$

where P denotes the parity of the permutation l of the index set $k(J)$. Hence, $|\langle J|a|K\rangle|^2$ describes the overlap of Slater determinants for different dot occupation ($J : n = 0$ and $K : n = 1$), which becomes non-trivial for $V \neq 0$.

As can be seen from Eq. (16), the original quantum dot level at E_0 is modulated by 2DEG-induced energy shifts $E^1_K - E^0_J$. Since $E^1_K - E^0_J$ (see Eqs. (13),(14)) vanishes for those J, K where $|\langle J|a|K\rangle|^2 \neq 0$ in the *non-interacting* case, we obtain $A(\omega) = 2\pi\hbar\delta(\hbar\omega - E_0)$ for $V = 0$ (note that the sum over all w is normalized and $U = \mathbf{1}$ w.o.l.g. in this case). For the *interacting* case, however, the sum over δ-peaks with weights $(w_J + w_K)|\langle J|a|K\rangle|^2$ and varying energy shifts $E^1_K - E^0_J$ may provide an overall shift (i.e., renormalization) of E_0 and a broadening of the δ-peak of the non-interacting case, depending on $k_B T$ and μ via w_J, w_K. A level broadening results, if $E^1_K - E^0_J$ has not the same value for all J, K with $(w_J + w_K)|\langle J|a|K\rangle|^2 \neq 0$. From a different perspective, the quantum dot electron experiences not only a classical confinement potential but also the quantum fluctuations from the non-classical term $\sum_{k,k'} v_{kk'} b^{\dagger}_k b_{k'}$ in H (see Eq. (2)). A quantitative estimation of the expected level broadening will be published elsewhere.

IV. POSSIBLE CONSEQUENCES FOR CHARGE QUBITS

As a consequence, if coupled quantum dot systems are employed as *charge* based qubits that are controlled by external metallic gate electrodes, the discussed level broadening mechanism might imply a reduction of the qubit phase coherence time. Even worse, if multiple, spatially separated gates are used, the electron position could become "macroscopically visible" in terms of measurable image charges inside the gate electrodes. In simple words, the spatially resolved multi-gate contacts make the charge position visible to the rest of the world in terms of detectable charges and currents through attached cables. This is a typical situation of marcoscopically distinguishable quantum states, leading to a mixed state of the reduced density matrix of the qubit system (due to entanglement with the external experimental setup, coupled via gate electrodes). Such a mechanism can also lead to a destruction of entanglement in qubits that are based on the spatial (i.e., charge) degree of freedom. In order to address these two limitations, superconducting gate electrodes (in particular, additional screening gates) might be used as a possible solution. In addition, the latter gate electrodes might also be bridged internally without any visible effect to the outside world. The important point is to avoid any entanglement of the qubits with the environment during quantum computation. *Spin* based qubits[10] appear to be more robust against the discussed gate effects.

V. SUMMARY

We have considered a 2D electron reservoir and its influence on a spatially separated quantum dot system due to a direct Coulomb interaction. As a main result, the dissipative reservoir may introduce level broadening to the quantum dot system, which can be understood in terms of a Coulomb scattering effect (due to dissipation of gate electrons). Consequences for charge qubit systems were discussed.

Acknowledgments

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